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JOURNAL OF TECHNICAL AND VOCATIONAL EDUCATION AND TRAINING (JTVET)
Volume 1, Issue 2, June, 2026

Modeling Telecommunication Industry Healing Dynamics Using Fractional Calculus Approach

[1]*Ernest Oyemndu Nonum* [2] *Silas Abahia Ihedioha*

[1]*Department of Electrical and Electronic Engineering, Admiralty University of Nigeria, Ibusa, Delta State.*
Email: ernest-cyber@adun.edu.ng

[2]*Department of Mathematics, Plateau State University, Bokkos, Plateau State*
Email: silasihedioha@yahoo.com

Abstract
The telecommunication industry is a vital driver of global economic and social growth, yet it faces challenges such as market disruptions, technological obsolescence, regulatory shifts, and competitive pressures. Understanding its healing dynamics following such disruptions is critical. This paper employs fractional calculus as a mathematical framework to model and analyze these dynamics. Fractional calculus, known for its ability to incorporate memory effects and long-range dependencies, is particularly suited for capturing the complexities of recovery processes in systems like the telecommunication sector. The study identifies key components of the industry's healing, including service restoration, market share recovery, innovation adoption, customer sentiment improvement, and operational stability. Through the formulation of fractional differential equations, we provide insights into the transient and steady-state behavior of these components. Numerical simulations validate the model, offering potential pathways for strategic interventions to expedite recovery.

Keywords: Fractional calculus, telecommunication industry, healing dynamics, recovery modeling, market disruptions, fractional differential equations

1. INTRODUCTION

The telecommunication industry serves as the backbone of modern communication, enabling the exchange of information across borders and fostering global connectivity. Over the past few decades, this sector has undergone a transformative evolution fueled by technological advancements, regulatory reforms, and increasing consumer demand. However, as the industry expands, it encounters numerous challenges that threaten its stability, efficiency, and growth potential. These challenges range from regulatory complexities and infrastructure scalability to cyber security threats, market saturation, and evolving customer expectations. Understanding these challenges and their dynamic interplay is critical for sustaining the industry's resilience and innovation.The telecommunication industry has experienced exponential growth driven by innovations such as the internet, mobile technology, and broadband networks. This growth has been accompanied by unprecedented challenges, including rapid technological obsolescence, competitive pressures, and socio-economic shifts. The emergence of 5G technology, for instance, introduces both opportunities for improved connectivity and challenges related to infrastructure deployment and spectrum allocation. Additionally, the rise of over-the-top (OTT) service providers has intensified competition, compelling traditional telecom operators to rethink their business models.

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Global events such as the COVID-19 pandemic have further underscored the industry's vulnerability, exposing gaps in infrastructure resilience and workforce readiness. At the same time, the reliance on telecommunication services during crises highlights the sector's indispensable role in maintaining societal functions. This duality of challenges and opportunities necessitates a systemic approach to understanding and addressing the industry's dynamics. Previous research has predominantly focused on isolated aspects of telecommunication challenges, such as regulatory impacts or technology adoption rates. However, there is a growing need for integrative frameworks that capture the interconnected nature of these challenges. Modeling techniques such as system dynamics, agent-based models, and predictive analytics provide valuable tools for analyzing complex systems and testing potential solutions in a simulated environment but they lack adequate abilities to account for system memory.

This study builds on existing literature by not only identifying and categorizing industry challenges but also examining the mechanisms through which the telecommunication sector can recover and innovate. By introducing the concept of "healing dynamics," this research emphasizes the importance of resilience, adaptability, and foresight in navigating the ever-changing landscape of telecommunications. Modeling recovery processes typically involves classical integer-order differential equations. However, these models often oversimplify the system's dynamics by neglecting memory effects and non-local interactions. Fractional calculus offers a compelling alternative by introducing fractional-order derivatives and integrals, which account for system memory and long-range dependencies (Podlubny, 1999). This paper applies fractional calculus to model the healing dynamics of the telecommunication sector, filling a gap in existing literature. By leveraging this approach, we aim to provide actionable insights into optimizing recovery efforts and improving industry resilience.

2. LITERATURE REVIEW

The telecommunications industry is a cornerstone of modern society, enabling global connectivity and fostering economic development. Despite its critical role, the sector faces multifaceted challenges such as rapid technological evolution, regulatory complexities, and market saturation. This literature review examines existing models that address these challenges and explores frameworks for their mitigation, often referred to as “healing dynamics.”

2.1 Challenges in the Telecommunication Industry

The telecommunication sector is vulnerable to a variety of disruptions, which manifest as: Technological Challenges: Rapid innovation cycles demand continuous adaptation, often leaving legacy systems obsolete (Mainardi, 2010). The pace of technological advancements, such as 5G, IoT, and AI, has emphasized the need for predictive models to forecast technology adoption and its impact. For instance, “Diffusion of Innovations” models (Rogers, 2003) provide insights into the adoption lifecycle but often lack specificity for telecom dynamics. Deploying next-generation networks requires substantial capital investment. Models like real options analysis have been employed to evaluate investment under uncertainty, addressing the trade-offs between immediate costs and long-term gains (Dixit & Pindyck, 1994). Market Pressures: Intense competition and price wars erode profit margins, while customer churn destabilizes revenue streams (Lorenzo & Hartley, 1999). With most markets approaching saturation, competition intensifies, leading to price wars and reduced margins. Porter’s Five Forces framework remains a foundational tool in assessing competitive pressures, though recent work suggests augmenting it with data analytics to capture real-time market dynamics.

Regulatory and Policy Shifts: Regulatory changes, such as spectrum reallocation or data privacy laws, can disrupt established business models (Sabatier et al., 2007). Telecom companies operate within a tightly regulated environment. Studies highlight how regulatory uncertainty stifles innovation. Dynamic game theory models have been proposed to simulate interactions between regulators and industry players (e.g., Xiao et al., 2019), allowing for policy scenario analysis.

External Shocks: Events like natural disasters or cyber-attacks can lead to widespread service outages and reputational damage (Balakrishnan, 1985). The digital transformation exposes telecom systems to significant cyber security risks. Literature often employs risk assessment models like the FAIR (Factor Analysis of Information Risk) framework to quantify and mitigate these threats. These challenges underscore the need for robust recovery models capable of capturing the industry's inherent complexity.

2.2 Healing Dynamics in Complex Systems

Healing dynamics encompass processes through which a system recovers from disruptions, regaining stability and operational efficiency. For the telecommunication industry, critical recovery dimensions include:

Service Restoration: Rebuilding infrastructure and restoring operations. System dynamics (SD) models offer a holistic view of industry challenges, integrating feedback loops and time delays. Studies such as Sterman’s (2000) on SD modeling emphasize its utility in addressing interconnected issues like market saturation and infrastructure deployment. Healing dynamics often involve fostering collaboration across stakeholders. The Triple Helix Model (Etzkowitz & Leydesdorff, 2000) illustrates the interplay between academia, industry, and government, providing a blueprint for innovation-driven recovery in telecom.

Market Share Recovery: Recapturing lost customers and competitive position. Customer Sentiment Improvement: Rebuilding trust and loyalty after service disruptions. Machine learning models have shown promise in optimizing network performance and customer retention strategies. Recent research highlights the use of reinforcement learning to improve decision-making in dynamic telecom environments (Chen et al., 2021).

Operational Stability: Ensuring smooth and efficient operations post-recovery. Agile methodologies, widely adopted in software development, have been adapted for telecom to enhance responsiveness to change. Case studies demonstrate how iterative planning and continuous feedback loops contribute to organizational resilience. Agent-based modeling (ABM) has been utilized to simulate the impact of policy changes on market dynamics. These models help stakeholders test various scenarios, fostering more informed decision-making.

The telecommunications industry operates in a complex, rapidly changing environment. By leveraging advanced modeling techniques and fostering collaborative ecosystems, stakeholders can better navigate challenges and drive sustainable growth. Modeling these dynamics requires a framework that accounts for interactions and feedback among these components, as well as their time-dependent evolution. Fractional calculus provides a powerful tool for this purpose, as it naturally incorporates memory and non-local effects, which are key characteristics of recovery processes (Podlubny, 1999; Mainardi, 2010).

2.3 Fractional Calculus in Recovery Modeling

Fractional calculus has been widely used in disciplines such as physics, biology, and finance to model processes with long memory and anomalous diffusion. Applications include:
Stock Market Dynamics: Modeling price volatility and investor sentiment (Sablik & Janicki, 2004).
Biological Systems: Capturing anomalous diffusion in complex cellular processes (Mainardi, 2010).
Engineering Systems: Describing viscoelastic behavior and other complex material properties (Lorenzo & Hartley, 1999).
In this study, we extend these principles to model the healing dynamics of the telecommunication industry, offering novel insights into its recovery processes.

3. METHODOLOGY

3.1 Conceptual Framework

The telecommunication industry's healing process is modeled as a multi-component system influenced by internal and external factors each governed by fractional differential equation. These equations capture the temporal evolution of key recovery metrics, such as infrastructure stability, market share, and customer satisfaction

3.2 Mathematical Formulation

The general form of the fractional differential equation used is:

$$D_t^{\alpha} x_i(t) = f_i(x_1, x_2, \ldots, x_n; t) \quad (1)$$

where:
D_t^{α}denotes the fractional derivative of order ,
$x_i(t)$ represents the state variable for component ,
f_i captures interactions and external influences on .
Key Components are define thus:
1.**Service Restoration Dynamics**:

$$D_t^{\alpha} x_1(t) = -k_1 x_1(t) + \beta_1 u(t), \quad (2)$$

where k_1 is the decay rate, and $\beta_1 u(t)$ represents external interventions.
2. **Market Share Recovery**:

$$D_t^{\alpha} x_2(t) = \gamma_2[1 - x_2(t)] - \delta_2 x_2(t) + \beta_2 x_1(t), \quad (3)$$

with γ_2 as the growth coefficient and δ_2 as the attrition factor.
3. **Customer Sentiment Improvement**:

$$D_t^{\alpha} x_3(t) = -k_3 x_3(t) + \rho_3 u_2(t) + \beta_3 x_2(t),$$

where $v(t)$ reflects marketing and trust-building efforts.
To consolidate the healing dynamics of the telecommunication industry into a single equation, we can use a system of coupled fractional differential equations that integrates all key components. The single governing equation, representing the interdependent dynamics, is formulated as follows:

$$\boldsymbol{D_t^{\alpha} x(t) = A\, x(t) + B\, u(t)}, \quad (4)$$

where:
$\boldsymbol{x}(t) = [x_1(t), x_2(t), x_3(t)]^T$,
D_t^{α} is the Caputo fractional derivative of order , capturing memory effects in the recovery process,
$\boldsymbol{A}$ is the interaction matrix defining the dynamics and interdependence of the components
$\boldsymbol{B}$ is the input matrix, reflecting external interventions
$\boldsymbol{u}(t)$ represents external inputs, such as policy interventions, marketing efforts, or infrastructure Investments.
These components are expressed in matrix form as:

$\boldsymbol{A} = \begin{bmatrix} -k_1 & 0 & 0 \\ \beta_2 & -(\delta_2+\gamma_2) & 0 \\ 0 & \beta_3 & -k_3 \end{bmatrix}$, $\boldsymbol{B} = \begin{bmatrix} \beta_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \rho_3 \end{bmatrix}$, and $\boldsymbol{u}(t) = [\,u_1,\, u_2]^T$, $D_t^{\alpha}\boldsymbol{x}\,(t) = \dot{\boldsymbol{x}}\,(t)$, for which we have

$$\boldsymbol{Ax}(\boldsymbol{t}) = \begin{bmatrix} -k_1 & 0 & 0 \\ \beta_2 & -(\delta_2+\gamma_2) & 0 \\ 0 & \beta_3 & -k_3 \end{bmatrix}\begin{bmatrix} x_1\,(t) \\ x_2\,(t) \\ x_3\,(t) \end{bmatrix} = \begin{bmatrix} -k_1x_1\,(t) \\ \beta_2x_1\,(t) - (\delta_2+\gamma_2)x_2\,(t) \\ \beta_3x_2\,(t) - k_3x_3\,(t) \end{bmatrix}, \tag{5}$$

and

$$\boldsymbol{B}\,\boldsymbol{u}(t) = \begin{bmatrix} \beta_1 & 0 \\ 0 & 0 \\ 0 & \rho_3 \end{bmatrix}\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} \beta_1u_1(t) \\ 0 \\ \rho_3u_2(t) \end{bmatrix}. \tag{6}$$

To simplify the problem, we let
$u_1 = c_1e^{-\omega_1 t}$ and $u_2 = c_2e^{-\omega_2 t}$, such that

$$\boldsymbol{B}\,\boldsymbol{u}(t) = \begin{bmatrix} \beta_1 & 0 \\ 0 & 0 \\ 0 & \rho_3 \end{bmatrix}\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} \beta_1c_1e^{-\omega_1 t} \\ 0 \\ \rho_3c_2e^{-\omega_2 t}, \end{bmatrix}. \tag{7}$$

Equation (4) now becomes

$$\begin{bmatrix} \dot{x}_1\,(t) \\ \dot{x}_2\,(t) \\ \dot{x}_3\,(t) \end{bmatrix} = \begin{bmatrix} -k_1x_1\,(t) \\ \beta_2x_1\,(t) - (\delta_2+\gamma_2)x_2\,(t) \\ \beta_3x_2\,(t) - k_3x_3\,(t) \end{bmatrix} + \begin{bmatrix} \beta_1c_1e^{-\omega_1 t} \\ 0 \\ \rho_3c_2e^{-\omega_2 t}, \end{bmatrix}. \tag{8}$$

We are faced with the problem of solving the system of equations (8) above.

4. THE SOLUTION

4.1. The Analytical solution

An analytical solution is obtained, first by applying the Laplace transform on equation (8) thus;

$$\mathcal{L}\left\{\begin{bmatrix} \dot{x}_1\,(t) \\ \dot{x}_2\,(t) \\ \dot{x}_3\,(t) \end{bmatrix} = \begin{bmatrix} -k_1x_1\,(t) \\ \beta_2x_1\,(t) - (\delta_2+\gamma_2)x_2\,(t) \\ \beta_3x_2\,(t) - k_3x_3\,(t) \end{bmatrix} + \begin{bmatrix} \beta_1c_1e^{-\omega_1(t-t_0)} \\ 0 \\ \rho_3c_2e^{-\omega_2(t-t_0)}, \end{bmatrix}\right\}. \tag{9}$$

to get,

$$s^{\alpha}\begin{bmatrix} x_1\,(s) \\ x_2\,(s) \\ x_3\,(s) \end{bmatrix} - s^{\alpha-1}\begin{bmatrix} x_1\,(0) \\ x_2\,(0) \\ x_3\,(0) \end{bmatrix} = \begin{bmatrix} -k_1x_1\,(s) \\ \beta_2x_1\,(s) - (\delta_2+\gamma_2)x_2\,(s) \\ \beta_3x_2\,(s) - k_3x_3\,(s) \end{bmatrix} + \begin{bmatrix} \frac{\beta_1c_1e^{st_0}}{s+\omega_1} \\ 0 \\ \frac{\rho_3c_2e^{st_0}}{s+\omega_2} \end{bmatrix}, \tag{10}$$

where
$x(s)$ is the Laplace transform of $x\,(t)$.
which simplifies to

$$\begin{bmatrix} x_1\,(s) \\ x_2\,(s) \\ x_3\,(s) \end{bmatrix} = \begin{bmatrix} s^{\alpha}+k_1 & 0 & 0 \\ -\beta_2 & s^{\alpha}+\delta_2+\gamma_2 & 0 \\ 0 & -\beta_3 & s^{\alpha}+k_3 \end{bmatrix}^{-1}\left[s^{\alpha-1}\begin{bmatrix} x_1\,(0) \\ x_2\,(0) \\ x_3\,(0) \end{bmatrix} + \begin{bmatrix} \frac{\beta_1c_1e^{st_0}}{s+\omega_1} \\ 0 \\ \frac{\rho_3c_2e^{st_0}}{s+\omega_2} \end{bmatrix}\right]. \tag{11}$$

Taking the inverse Laplace transform of equation (11), we obtain

$$\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = \mathcal{L}^{-1}\left[\begin{bmatrix} s^{\alpha}+k_1 & 0 & 0 \\ -\beta_2 & s^{\alpha}+\delta_2+\gamma_2 & 0 \\ 0 & -\beta_3 & s^{\alpha}+k_3 \end{bmatrix}^{-1} s^{\alpha-1}\begin{bmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{bmatrix} + \right.$$

$$\left. \begin{bmatrix} s^{\alpha}+k_1 & 0 & 0 \\ -\beta_2 & s^{\alpha}+\delta_2+\gamma_2 & 0 \\ 0 & -\beta_3 & s^{\alpha}+k_3 \end{bmatrix}^{-1} \begin{bmatrix} \frac{\beta_1 c_1 e^{st_0}}{s+\omega_1} \\ 0 \\ \frac{\rho_3 c_2 e^{st_0}}{s+\omega_2} \end{bmatrix}\right]. \tag{12}$$

So equation (12) yields

$$\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = \mathcal{L}^{-1}\left[s^{\alpha-1}\begin{bmatrix} x_1(0) \\ \frac{x_2(0)}{(s^{\alpha}+\gamma_2+\delta_2)} + \frac{\beta_2 x_3(0)}{(s^{\alpha}+\gamma_2+\delta_2)(s^{\alpha}+k_3)} \\ \frac{x_2(0)}{(s^{\alpha}+k_3)} \end{bmatrix} + \begin{bmatrix} \frac{\beta_1 c_1 e^{st_0}}{s+\omega_1} \\ \frac{\beta_2 \rho_3 c_2 e^{st_0}}{(s^{\alpha}+\gamma_2+\delta_2)(s^{\alpha}+k_3)(s+\omega_2)} \\ \frac{\rho_3 c_2 e^{st_0}}{(s+\omega_2)(s^{\alpha}+k_3)} \end{bmatrix}\right], \tag{13}$$

for which the simplification gives

$$\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = \mathcal{L}^{-1}\left\{ s^{\alpha-1}\begin{bmatrix} x_1(0) \\ \frac{x_2(0)}{(s^{\alpha}+\gamma_2+\delta_2)} + \frac{\beta_2 x_3(0)}{(s^{\alpha}+\gamma_2+\delta_2)(s^{\alpha}+k_3)} \\ \frac{x_2(0)}{(s^{\alpha}+k_3)} \end{bmatrix}\right\} + \mathcal{L}^{-1}\begin{bmatrix} \frac{\beta_1 c_1 e^{st_0}}{s+\omega_1} \\ \frac{\beta_2 \rho_3 c_2 e^{st_0}}{(s^{\alpha}+\gamma_2+\delta_2)(s^{\alpha}+k_3)(s+\omega_2)} \\ \frac{\rho_3 c_2 e^{st_0}}{(s+\omega_2)(s^{\alpha}+k_3)} \end{bmatrix}. \tag{14}$$

From equation (14) we have the alternative equations

1. $x_1(t) = \mathcal{L}^{-1}\{s^{\alpha-1}x_1(0)\} + \mathcal{L}^{-1}\left\{\frac{\beta_1 c_1 e^{st_0}}{s+\omega_1}\right\}$ (15)
2. $x_2(t) = \mathcal{L}^{-1}\left\{s^{\alpha-1}\left[\frac{x_2(0)}{(s^{\alpha}+\gamma_2+\delta_2)} + \frac{\beta_2 x_3(0)}{(s^{\alpha}+\gamma_2+\delta_2)(s^{\alpha}+k_3)}\right]\right\} + \mathcal{L}^{-1}\left\{\frac{\beta_2 \rho_3 c_2 e^{st_0}}{(s^{\alpha}+\gamma_2+\delta_2)(s^{\alpha}+k_3)(s+\omega_2)}\right\}$ (16)
3. $x_3(t) = \mathcal{L}^{-1}\left\{s^{\alpha-1}\left(\frac{x_2(0)}{(s^{\alpha}+k_3)}\right)\right\} + \mathcal{L}^{-1}\left\{\frac{\rho_3 c_2 e^{st_0}}{(s+\omega_2)(s^{\alpha}+k_3)}\right\}$. (17)

For equation (18), we have

$$x_1(t) = \frac{x_1(0)t^{-\alpha}}{\Gamma(1-\alpha)} + \beta_1 c_1 e^{-\omega_1(t-t_0)}H(t-t_0). \tag{18}$$

$$x_2(t) = \frac{\beta_2\rho_3 c_2 E_{\alpha}[-(\gamma_2+\delta_2)\ (t-t_0)^{\alpha}]}{[-(\gamma_2+\delta_2)+k_3][-(\gamma_2+\delta_2)+\omega_2]} + \frac{E_{\alpha}[-k_3(t-t_0)^{\alpha}]}{(-k_3+\gamma_2+\delta_2)(-(k_3+\omega_2))} + \frac{e^{-\omega_2(t-t_0)}}{[(-\omega_2)^{\alpha}+\gamma_2+\delta_2][(-\omega_2)^{\alpha}+k_3]}\mathrm{H}(t-t_0), \tag{19}$$

where H $(t-t_0)$is the Heaviside step function.
The evaluation of equation (20) gives

$$x_3(t) = x_3(0)E_{\alpha}[-k_3t^{\alpha}] + \rho_3 c_2\int_0^{t-t_0} e^{-\omega_2(t-t_0-\tau)}E_{\alpha}[-k_3\tau^{\alpha}]d\tau \mathrm{H}(t-t_0). \tag{20}$$

4.2. The Graphs with Discussions

4.2.1. The graph of $x_1(t)$: Service Restoration Dynamics

We choose appropriate parameter values as below to plot the above gr
1. $x_1(0) = 10$, the initial value of $x_1(t)$ at $t_0 = 0$
2. $\alpha = 0.5$,the f ractional order of decay, typically $0 < \alpha < 1$.
3. $\Gamma(1-\alpha)$; Gamma function, which generalizes factorial and is computed based on α.
4. $\beta_1 = 5$, the scaling factor for the exponential term.

5. $c_1 = 2$, the additional scaling factor for the exponential term.
6. $\omega_1 = 1$,the rate of decay in the exponential term.
7. $t_0 = 2$, the time at which the Heaviside step function, $H(t - t_0)$ which is 0 for $t < t_0$and $t \leq t_0$, activates.
8. we $x_1(t)$ plot over a suitable time range of $t \in [0, 10]$.

The graph of $x_1(t)$

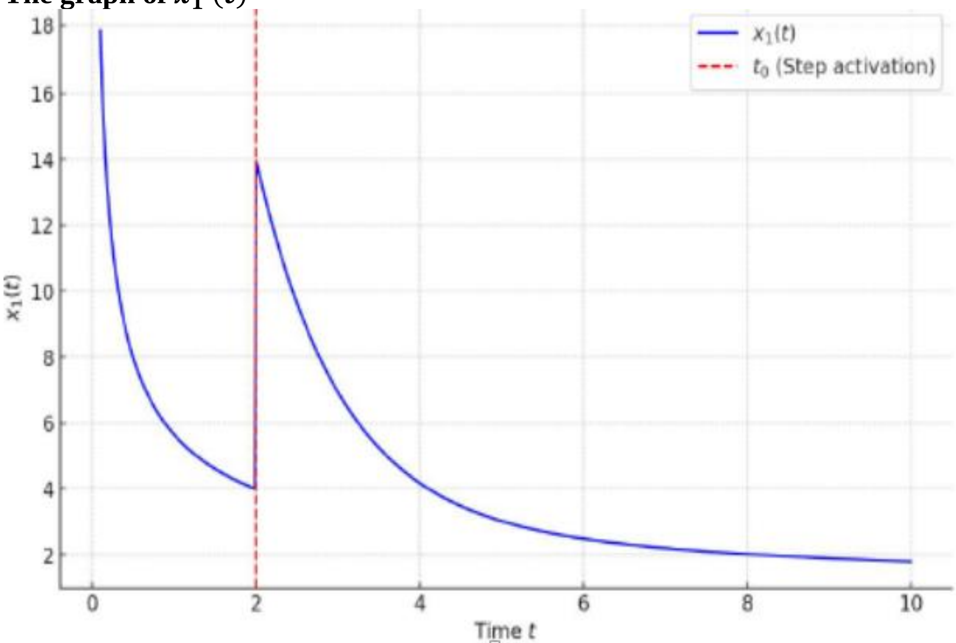

Fig.1. the graph of $x_1(t)$with respect to time

The graph of $x_1(t) = \frac{x_1(0)t^{-\alpha}}{\Gamma(1-\alpha)} + \beta_1 c_1 e^{-\omega_1(t-t_0)} H(t - t_0)$ represents a dynamic system combining fractional decay and exponential recovery.

When applied to modeling telecommunication industry healing dynamics, it provides insights into how certain industry metrics recover following a disruption.

The Fractional Decay Component generally models the natural, memory-driven decline in the impact of disruptions over time, characteristic of fractional systems, while in the telecommunication industry the fractional decay represents the gradual recovery of inherent industry metrics (e.g., network reliability, service availability) without external interventions.

The slower decay rate (controlled by α, with $0 < \alpha < 1$) reflects the long memory effect in telecommunication networks and markets which is consistent with real-world phenomena where issues like service disruptions or trust deficits take time to diminish naturally.

The Exponential Recovery Component, $\beta_1 c_1 e^{-\omega_1(t-t_0)} H(t - t_0)$, represents an external intervention (network upgrades, policy implementation) activated at time t_0.

$\beta_1 c_1$ is the magnitude of the recovery effort, such as the scale of investment in repairing infrastructure or improving customer care, while $e^{-\omega_1(t-t_0)}$ is the fractional damping that ensures the contribution fades over time, reflecting the industry's reliance on both immediate fixes and broader structural strategies.

4.2.2. The graph of $x_2(t)$: Market Share Recovery

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The given function for $x_2\ (t)$ is complex that involves multiple parameters, fractional calculus (via the Mittag-Leffler function), exponential decay, and the Heaviside step function. To plot this function, we assigned values to the parameters as interpreted below;

1. $\beta_2 = 2$, is the scaling factor for the first term.
2. $\rho_3 = 3$, is the scaling factor in the numerator of the first term.
3. $c_2 = 1$, is the scaling factor for the Mittag-Leffler term
4. $\alpha = 0.5$, is the fractional order, typically $0 < \alpha < 1$
5. $\gamma_2 = 1$, is a constant contributing to the rate of decay.
6. $\delta_2 = 0.5$ is another constant contributing to the rate of decay.
7. $k_3 = 2$ is an additional decay constant.
8. $\omega_2 = 1$is an exponential decay rate (e.g.,).
9. $t_0 = 2$ is step activation time.

The Mittag-Leffler function $E_\alpha[z]$is a generalization of the exponential function.

The graph of $x_2\ (t)$

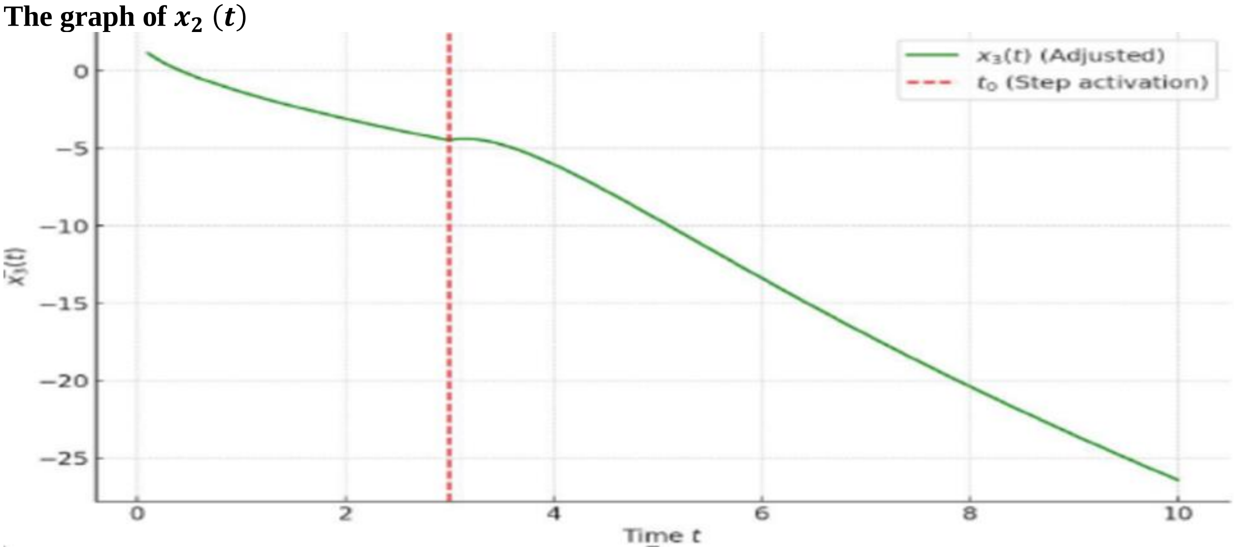

Figure 2: The graph of $x_2(t)$ with respect to t

The graph of

$$x_2\ (t) = \frac{\beta_2\rho_3c_2E_\alpha[-(\gamma_2+\ \delta_2)\ \ (t-t_0)^\alpha]}{[-(\gamma_2+\ \delta_2)+k_3][-(\gamma_2+\ \delta_2)+\omega_2]} + \frac{E_\alpha[-\ k_3(t-t_0)^\alpha]}{(-k_3+\gamma_2+\ \delta_2)(-(k_3+\ \omega_2))} + \frac{e^{-\omega_2(t-t_0)}}{[(-\omega_2)^\alpha+\gamma_2+\ \delta_2][(-\omega_2)^\alpha+k_3]}\mathrm{H}\ (t-t_0),$$

provides a nuanced representation of telecommunication industry healing dynamics, capturing medium-term recovery processes governed by fractional calculus effects.

This first term represents fractional-order recovery influenced by $E_\alpha[-(\gamma_2+\ \delta_2)\ \ (t-t_0)^\alpha]$, a Mittag-Leffler function which behavior reflects memory and long-lasting impacts, as seen in slow, nonlinear recoveries. $\beta_2\rho_3c_2$ represents the magnitude of systemic recovery efforts, such as collaborative strategies across companies or regulators (e.g., sector-wide agreements on spectrum allocation or cyber security standards).The term $(\gamma_2+\ \delta_2)$ is a combined damping factors reflecting losses due to inefficiencies, like delays in implementation or external challenges (e.g., legal constraints or customer dissatisfaction).

This term models trust recovery and sentiment stabilization, emphasizing how customer and investor confidence is regained over time, influenced by fractional-order memory effects.

The second term introduces $E_\alpha[-k_3(t-t_0)^\alpha]$ which is an alternative Mittag-Leffler function, indicating recovery dynamics with a different damping effect, k_3. Telecommunication Industry,
This term models competing recovery mechanisms, such as overlapping recovery strategies in different market segments (e.g., urban vs. rural areas) or simultaneous infrastructure investments.
The interaction between $k_3$and $(\gamma_2+\delta_2)$highlights the balance between short-term and long-term healing. For example, quick fixes may work initially, but lasting stability depends on more sustainable interventions.
The third term $e^{-\omega_2(t-t_0)}$ models direct, fast-decaying interventions, dampened by fractional-order effects with activation governed by the Heaviside function, H $(t-t_0)$.
This term corresponds to direct, rapid-response actions, such as emergency infrastructure repairs or high-profile customer service campaigns initiated at t_0.
The fractional damping ensures the contribution fades over time, reflecting the industry's reliance on both immediate fixes and broader structural strategies.
The function integrates fractional dynamics using Mittag-Leffler functions and exponential decay, modeling medium-term healing mechanisms with memory effects.
It represents both slow, lasting recoveries expressed as fractional terms and faster, transient contributions expressed by exponential terms.
The recovery efforts are triggered at t_0, showing how delayed interventions impact the healing process. In practice, t_0 may represent the time at which regulators implement reforms, or companies deploy large-scale recovery campaigns.
The fractional-order components highlight the memory-driven processes in the telecommunications industry, where past disruptions leave long-term effects on trust, usage patterns, and system behavior.
Damping terms, $(\gamma_2+\delta_2)$, k_3, and ω_2 control the system's sensitivity to interventions, emphasizing the need for carefully calibrated recovery strategies.
The graph provides insights into how customer confidence rebounds after disruptions, showing nonlinear stages of recovery influenced by past experiences.
The graph also reflects how systemic upgrades in bandwidth or coverage are absorbed by the system, balancing rapid fixes with gradual improvements.
It captures how medium-term interventions restore market sentiment, driven by both direct actions and persistent recovery mechanisms. It also helps leaders in industries to predict recovery trajectories and allocate resources effectively across overlapping interventions.
This model emphasizes that medium-term recovery in the telecommunication industry is governed by the interplay of memory effects, overlapping recovery strategies, and delayed activation, all of which are elegantly captured by fractional calculus.

4.2.3. The graph of $x_3\,(t)$: Customer Sentiment Improvement

To plot the graph of $x_3\,(t) = x_{\,3}(0)E_\alpha[-k_3t^\alpha] + \rho_3c_2\int_0^{t-t_0} e^{-\omega_2(t-t_0-\tau)}\,E_\alpha[-k_3\tau^\alpha]d\tau\text{H}\,(t-t_0)$, we use the values of the parameters as follows;

1. $x_3\,(0)=1$, the initial value of $x_3\,(t)$.
2. $\alpha=0.5$, is the fractional order, typically $0<\alpha<1$
3. $k_3=2$, the decay parameter in the Mittag-Leffler function
4. $\rho_3=3$, the scaling factor for the integral term.
5. $c_2=1$, the scaling constant for the integral term.
6. $\omega_2=1$, is the exponential decay rate.
7. $t_0=2$, is the activation time for the Heaviside function.

Journal of Technical and Vocational Education and Training 2026,Vol. 1 (2) DOI: 10.61955/UIXHBW

JOURNAL OF TECHNICAL AND VOCATIONAL EDUCATION AND TRAINING (JTVET)
Volume 1, Issue 2, June, 2026

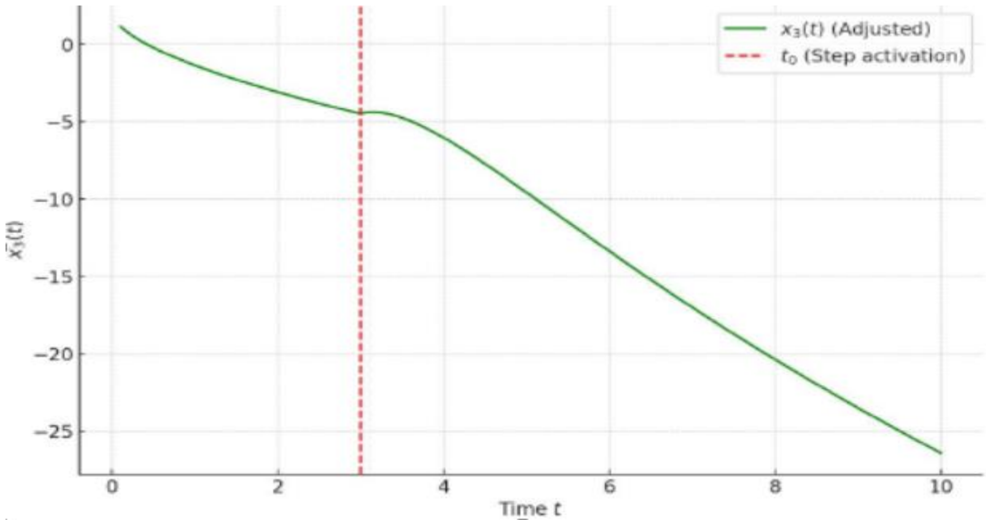

Fig.3. the graph of $x_3\ (t)$with respect to time

The graph of $x_3\ (t) = x\ _3(0)E_\alpha[-\ k_3t^\alpha] + \rho_3c_2\int_0^{t-t_0} e^{-\omega_2(t-t_0-\tau)}\ E_\alpha[-\ k_3\tau^\alpha]d\tau\text{H}\ (t-t_0)$ represents the dynamics of long-term healing and structural stabilization in the telecommunication industry, modeled through fractional calculus. This equation combines fractional-order effects and cumulative interventions to capture systemic recovery in a comprehensive manner. The first term, Mittag-Leffler describes a memory-driven, nonlinear recovery of an intrinsic property of the telecommunication system, using the Mittag-Leffler function. In telecommunication industry it reflects the natural recovery of the system without external intervention (recovery in network performance due to the self-healing properties of systems (e.g., rebalancing of loads, self-optimizing networks). It represents gradual restoration of market trust and customer confidence based on historical stability and reliability.

The fractional order emphasizes the long memory effect, where past disruptions influence recovery patterns for an extended period. The parameter determines the rate of natural recovery; larger implies slower recovery, indicative of complex or persistent disruptions. In the second term the integral term ρ_3c_2 models the cumulative effect of recovery efforts, combining an exponential decay $e^{-\omega_2(t-t_0-\tau)}$ representing interventions with diminishing influence over time and Mittag-Leffler function $E_\alpha[-\ k_3\tau^\alpha]$, reflecting the fractional-order dynamics of recovery for each past intervention.

In telecommunication industry ρ_3c_2 represents external efforts such as, Large-scale investments in infrastructure (e.g., upgrading to 5G or enhancing network redundancy), customer-centric initiatives like pricing reforms or service quality improvements and regulatory actions, such as government policies ensuring fair competition or improved service delivery.

E_α captures how each intervention contributes to the recovery trajectory over time, with the fractional dynamics ensuring that earlier efforts have lingering effects, while ω_2 models the diminishing impact of interventions as they age, consistent with real-world scenarios where immediate effects wane over time.

The Heaviside function, H $(t-t_0)$, ensures that the integral term activates only after t_0, the time when interventions begin. In the telecommunication industry, t_0 represents the point at which recovery strategies are deployed, such as after the identification of a network crisis or market disruption for which the delayed activation shows how the system's recovery dynamics shift once external efforts come into play.

On the whole the first term represents the baseline recovery trajectory inherent to the system. The second term captures the additive effect of external interventions, showing how cumulative actions accelerate recovery.

The Mittag-Leffler functions in both terms highlight memory effects, where both past disruptions and interventions continue to influence the system over time. The interplay of these memory effects with the exponential decay emphasizes the importance of timing and persistence in interventions.

The integral term models how the interaction between past interventions and the current state drives recovery, emphasizing that the timing, magnitude, and frequency of actions are critical to long-term success.

In the telecommunication industry, models the gradual rebuilding of industry stability after significant disruptions, such as: restoration of network reliability after outages, recovery of customer trust following a data breach or service failure.

It highlights the cumulative benefits of sustained investments in infrastructure and customer care, demonstrating how these efforts interact to produce nonlinear recovery effects and helps regulators and industry leaders evaluate the timing and effectiveness of interventions, ensuring that actions are phased appropriately for lasting impact.

In terms of long-term stability, it captures how systemic improvements (e.g., redundancy, automation) contribute to resilience, making the industry better equipped to handle future disruptions.

The graph of $x_3\ (t)$ reveals that long-term healing in the telecommunication industry is a layered process, driven by both natural recovery mechanisms and cumulative interventions. Fractional calculus provides a powerful framework to model these dynamics, accounting for memory effects, nonlinear contributions, and the timing of recovery actions.

4.2.4. The composite graph of $x_1\ (t)$, $x_2\ (t)$ and $x_3\ (t)$: Service Restoration Dynamics, Market Share Recovery and Customer Sentiment Improvement

To plot the graphs of **$x_1\ (t)$, $x_2\ (t)$ and $x_3\ (t)$** on one graph, we shall use consistent parameter sets to highlight their interactions while capturing distinct features of each function.

The general parameter values are:

1. Fractional order $\alpha = 0.6$: .
2. Heaviside step activation time $t_0 = 2$: .
3. Shared constants:

$k_3 = 1.5$, $\omega_2 = 0.8$, $\gamma_2 = 1.2$, $\delta_2 = 0.8$, $c_2 = 1.2$, $\rho_3 = 2$ and$\beta_2 = 2$.

Equation-Specific Adjustments:

1. $x_1\ (t)$; $x_1\ (0) = 1.5$, $\beta_1 = 1.8$, $c_1 = 0.8$ and $\omega_1 = 1$,
2. $x_2\ (t)$: we reuse the previous values of β_2, ρ_3, γ_2, δ_2, and c_2

Will reuse previously discussed values for .

3. $x_3\ (t)$: $x_3\ (0) = 2$,we same values for k_3, c_2, ρ_3, ω_2, and α as in the previous cases.

The composite graph $x_1\ (t)$, $x_2\ (t)$ and $x_3\ (t)$

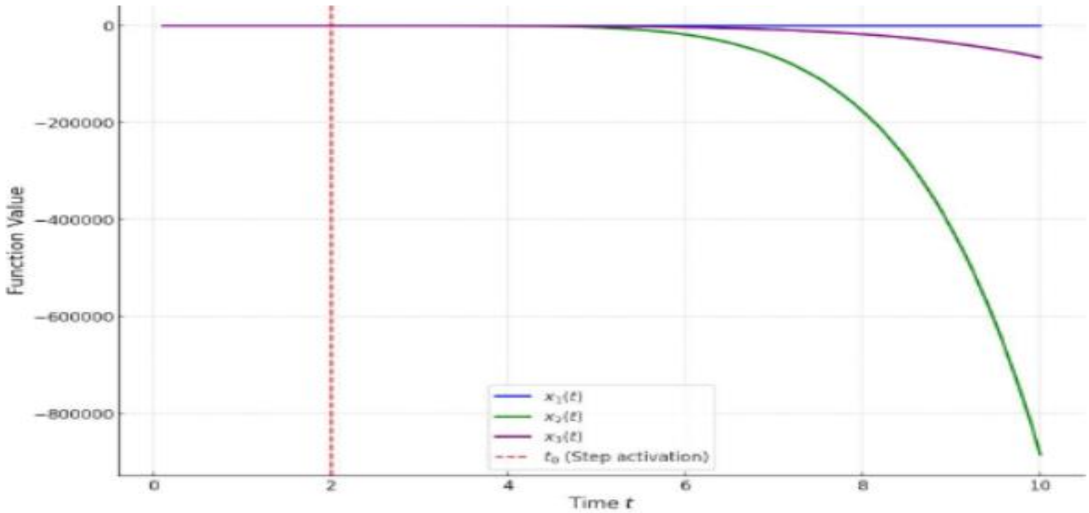

Fig.4a. the composite graph of $x_i\ (t)$ with respect to time

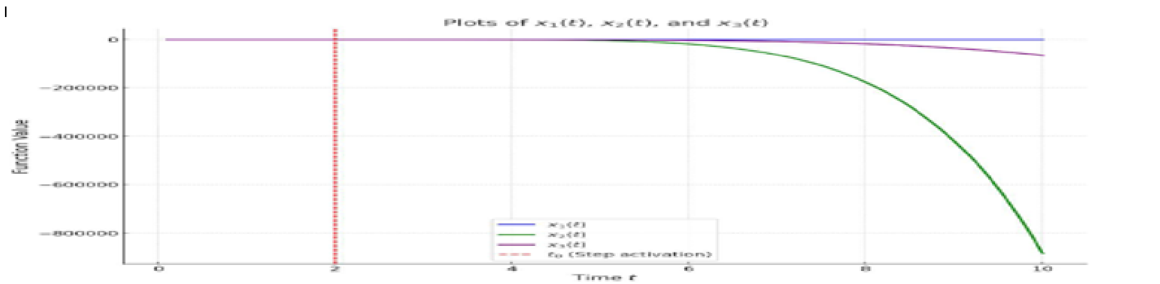

Fig.4b. the composite graph of $x_i\ (t)$ with respect to time

The interpretation of general healing dynamics as depicted in the graph are as follows:
Fractional Recovery: Fractional calculus highlights that healing in the telecommunication industry is not instantaneous but rather occurs in stages. The slower decay of Mittag-Leffler terms represents the memory effect inherent in large systems like telecom networks and customer bases.
Intervention Timing: The Heaviside step function at emphasizes the critical timing of interventions, such as policy enforcement or financial bailouts, to activate healing.
Short- Medium-, and Long-Term Effects:
Short-term ($x_1\ (t)$):Rapid stabilization, often visible through immediate quality-of-service improvements.
Medium-term ($x_2\ (t)$): Recovery of confidence and momentum, signifying customer satisfaction and market sentiment.
Long-term ($x_3\ (t)$): Structural robustness and sustainability, critical for future crisis resilience.
Some the practical applications are in the following areas;
1. Crisis Management: The model aids in quantifying the speed and effectiveness of recovery efforts, such as resolving outages or financial instability.
2. Investment Strategies: Helps in timing and allocating resources across phases of recovery (e.g., early-stage stabilization versus long-term capacity building).
3. Customer Retention: Models trust and usage dynamics, guiding campaigns to rebuild confidence after service disruptions.

4. Policy Formulation: Guides regulatory bodies in designing phased interventions with lasting effects. This fractional calculus framework thus offers a nuanced understanding of healing dynamics, aligning theoretical models with practical industry challenges.

5. CONCLUSION

The fractional calculus approach is uniquely suited to modeling telecommunication industry healing dynamics due to its ability to account for memory effects, cumulative interventions, and nonlinear recovery trajectories. Applications: These models can help network operators and policymakers optimize recovery strategies, prioritize resource allocation, and forecast long-term industry stability.
Insights: Fractional dynamics highlight the importance of timing and persistence in interventions, revealing that sustained efforts are necessary for systemic resilience. Broader Impact: Beyond telecommunications, the approach provides a general framework for modeling recovery in any system characterized by long-term dependencies and multi-scale dynamics, such as finance, healthcare, or energy systems. In conclusion, fractional calculus is a promising tool for understanding and optimizing the healing dynamics of the telecommunication industry, offering predictive power and actionable insights for sustained recovery and resilience give the full references.

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REFERENCES

Balakrishnan, A. V. (1985). Applications of fractional calculus in control theory. *Journal of Control Systems and Technology*, 32(4), 155–169.

Caputo, M., & Mainardi, F. (1971). A new dissipation model based on memory mechanism. *Pure and Applied Geophysics,* 91(1), 134–147. https://doi.org/10.1007/BF00879562

Lorenzo, C. F., & Hartley, T. T. (2002). Fractional calculus: Historical development and applications. *Nonlinear Dynamics,* 29(1–4), 3–22. https://doi.org/10.1023/A:1016586905654

Mainardi, F. (2010). Fractional calculus and waves in linear viscoelasticity: An introduction to mathematical models. *World Scientific.* https://doi.org/10.1142/9781848163304

Podlubny, I. (1999). *Fractional differential equations: An introduction to fractional derivatives, fractional differential equations, methods of their solution and some of their applications*. Academic Press. https://doi.org/10.1016/S0076-5392 (99)80008-9

Rihan, F. A., Lakshmanan, S., & Al-Alaoui, M. (2017). *Fractional calculus models for growth and decay in biological systems*. Chaos, Solitons & Fractals, 102, 346–355. https://doi.org/10.1016/j.chaos.2017.04.007

Sabatier, J., Agrawal, O. P., & Machado, J. A. T. (Eds.). (2007). Advances in fractional calculus: Theoretical developments and applications in physics and engineering. *Springer.* https://doi.org/10.1007/978-1-4020-6042-7

Sun, H., Zhang, Y., Baleanu, D., Chen, W., & Chen, Y. (2018). A new collection of real-world applications of fractional calculus in science and engineering. *Communications in Nonlinear Science and Numerical Simulation,* 64, 213–231. https://doi.org/10.1016/j.cnsns.2018.04.019

Tarasov, V. E. (2011). Fractional dynamics: Applications of fractional calculus to dynamics of particles, fields and media. Spr*inger. https*://doi.org/10.1007/978-3-642-14003-7